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BIKE TECH

Bicycling Magazine's Newsletter for the Technical Enthusiast

June 1985

Volume 4, Number 3

IN THE LAB

Stiffness Characteristics of Bicycle Wheels

Dan Price with Arthur Akers

Dan Price has been building wheels for the past six years, most recently in the shop of Mr. Jasjit Grewal of Sherpa Sports, Ltd., Aspen, Colorado. He built many of the wheels used by Jasjit's son, Alexi Grewal, in the 1982 and 1983 racing seasons. The tests reported here were performed at Iowa State University (Ames, Iowa) as part of Dan's work for a B.S. degree in Mechanical Engineering. Dan now works as Product Design Engineer at Osmonics, Inc. (Minnetonka, Minnesota).

Arthur Akers is Associate Professor of Engineering Science and Mechanics at Iowa State University, where he specializes in tribology, fluid power, and design.

How does spoke pattern influence the stiffness of spoked wheels? Specifically, how is stiffness in the torsional, lateral, and radial directions affected by changes in the spoking pattern? Do the various stiffnesses interact with each other?

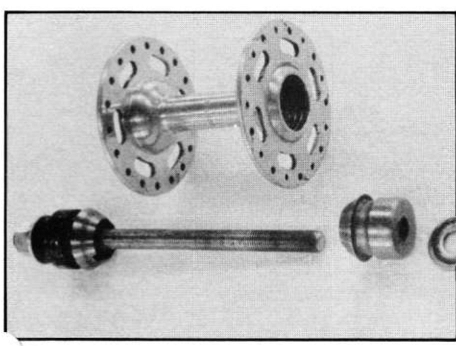

Figure 1: Hub mounting spindle made from Bendix coaster-brake hub parts.

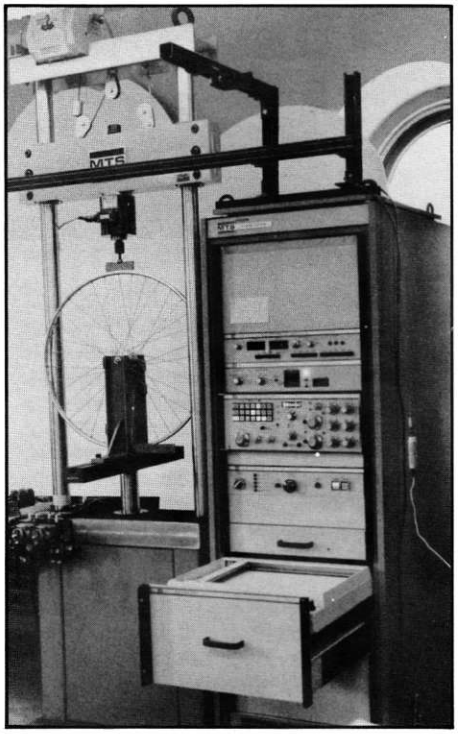

Figure 2: The MTS machine (with radial test fixture) showing control panel, load application device, and recorder.

Stiffness is defined as the force required to produce a given deflection. (The *directions* of the force and deflection must be specified.) Some writers on wheel building suggest that strength, not stiffness, is the most important practical property of a bicycle wheel. Despite this, many wheel builders often question whether certain spoking patterns produce greater energy losses as a result of wheel flexing. We wanted to answer these questions in relation to building wheels for competitive use, especially in view of the surprising lack of empirical data on the subject.

We felt that simple laboratory tests would give the most straightforward answers. Fortunately, we have access to a precision tensile testing machine, and a well-equipped shop for fabricating test jigs and fixtures. With these facilities, we tested wheels with five different types of spoking patterns (radial through 4-cross) for torsional, lateral,

IN THIS ISSUE

COMING IN THE AUGUST ISSUE

The Browning Automatic Bicycle Transmission: How it works, and How it is made.

and radial stiffness. The results generally confirm some long-standing articles of faith of the wheel building trade. At the same time, they raise certain questions about wheel stiffness that builders should know about.

The Wheels

All the tests used wheels built with the same materials, components, and spoke tension. The components are:

—hubs: 36-hole Normandy standard hi-flange front hubs
—spokes: DT 15-gage (1.8 millimeter) 18/8 stainless steel straight-gauge spokes
—rims: AVA aluminum-alloy 700c tubular rims (approx. weight 420 gm)

A spoke tensiometer was used to ensure an average spoke tension of 136 lb. (605 Newtons) in all the wheels. This standardization minimizes random errors in testing.

Five differently spoked wheels were built, using a radial, one-, two-, three-, and four-cross spoke pattern, with the patterns defined as the number of spokes crossed by a single spoke as it goes from the hub to the rim. On all wheels, a mirror-image spoke pattern was used; the spoke interlacing occurred at the crossing nearest to the rim.

To eliminate deflections caused by the ball bearings and the axle, the bearings and small diameter front axles were removed and replaced with a pair of hardened steel cones mounted on a 3/8 inch Bendix coaster-brake

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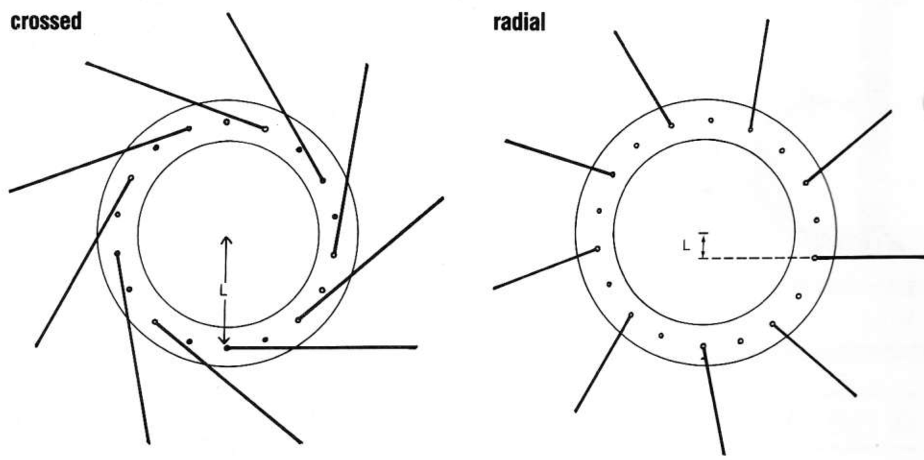


Figure 3: Torque stiffness of crossed versus radial spoking. L = lever arm distance

axle (Figure 1). These cones were tightened snugly into the hub shell. The assembly provided a way to support the wheel in a rigid manner so that the loads and deflections measured were those related to the wheel alone.

Test Fixtures

We were looking only at wheel stiffness, so deflections of the wheel-holding fixtures had to be eliminated. Accordingly, the test apparatus was carefully designed and constructed of very stout materials. During each test, deflections of the test apparatus were measured and found to be negligible, since the forces used in the stiffness tests were much lower than those which would have produced significant deflections in the test apparatus itself.

The radial and lateral tests were both performed with a Material Testing Systems (MTS) electro-hydraulic testing machine in the Mechanics Department lab at Iowa State University (Figure 2). The MTS machine consists of a large vertical jack operated by an electro-hydraulic servo-valve and a hydraulic pump. Its greatest asset is its ability to automatically plot load and displacement in an analog manner on an x-y flatbed plotter.

Torsional Stiffness

Torsional stiffness is defined as the torque required to produce unit angular rotation of the hub with respect to the rim. For rear wheels, greater torsional stiffness means that less "wind-up" of the hub occurs with a given pedaling torque. Torsional stiffness is strongly influenced by the hub shell diameter and the spoke pattern.

A larger hub shell diameter will produce a torsionally stiffer wheel because it increases the lever arm distance through which the spokes act on the hub. With torque remaining constant, spoke tension must necessarily decrease, resulting in less elongation of the spokes.

The spoking pattern has a much greater effect on torsional stiffness than does hub

Editor's note: Reaction to Leonard Goldberg's new book The Spoking Word *(reviewed in April 1985* Bike Tech*) ranged from amusement to appreciation. One writer stated "I loved your April Fool article on spoke patterns; whoever built that wheel would do anything for a laugh." Another asked for "more on wheelbuilding; it's still too much of an art and not enough of a science." In any case, there's continuing interest in novel designs and components for spoked wheels.*

Recent developments: Spoked wheels almost became obsolete overnight when disk wheels proved their aerodynamic superiority in international competition. But thanks to a ruling by the Union Cycliste Internationale (UCI), spoked wheels will be with us for a long time to come. The UCI declared, at their August 1984 meeting in Barcelona, that all wheels for future UCI-sanctioned events must have 16 to 40 spokes and must be at least 60 cm in diameter. Curiously, the UCI has refused requests to clarify whether disk wheels are outlawed by the "spoke rule". Disk wheel advocates may have to rest content with this tacit approval of their efforts. Meanwhile, the UCI decision (or indecision) is motivating renewed efforts to improve the design of spoked wheels. Optimistic wheelbuilders in the U.S. assert that spoked wheels can be made to have as little drag as disk wheels; they cite the combined benefits of rims with "aero" cross-section, thin airfoil spokes, novel arrangements with minimum spokes, and other tricks. Future issues of Bike Tech *will cover these developments.*

In the article presented here, the authors report on their careful laboratory measurements of wheel stiffness, and help relieve the shortage of empirical data on this topic. Until now, we had to rely mainly on calculations, computer simulations, and our own subjective impressions, all subject to notorious limitations. The authors' tests bypass these problems and, while they don't shatter any myths, they do provide the necessary solid baseline for comparison of new designs. In the sidebar following, we compare these lab results against the calculations given in two popular books on bicycle wheel design.

diameter. The reason is that small changes in the spoke's placement within the wheel have a large effect on the lever-arm distance (L in Figure 3) measured perpendicularly from the spoke's line of action to the hub center. Thus, 4-cross wheels (in which spokes are tangent to hub in 36 spoke wheels) are expected to be very stiff in torsion, while 0-cross wheels (radial spokes) should have almost zero torsional stiffness (since their lever-arm distance is, in theory, zero).

A consistent testing procedure was used to measure the loads and angular rotations during each test. Each hub was securely mounted in the test fixture (Figure 4) with a device to prevent it from rotating (Figure 5). A precision dial indicator gauge was positioned to measure the amount of rim travel tangential to the rim curvature (Figure 6). All wheels were built and mounted in the fixture so that the torque was transmitted through inside pulling spokes.

With the hub shell held stationary, torque was applied to the rim by means of a cable. The cable was fastened inside the valve stem hole and then wrapped approximately one full turn around the outside of the rim continuing vertically downward to the applied weights. The weights were calibrated for use as scale counterbalances. The radius to the line of action of the cable and the radius to the dial indicator were carefully measured so that the angle of rotation of the rim could be calculated accurately. Loads varied from zero to about 50 pounds, and were added carefully to avoid dynamic loading.

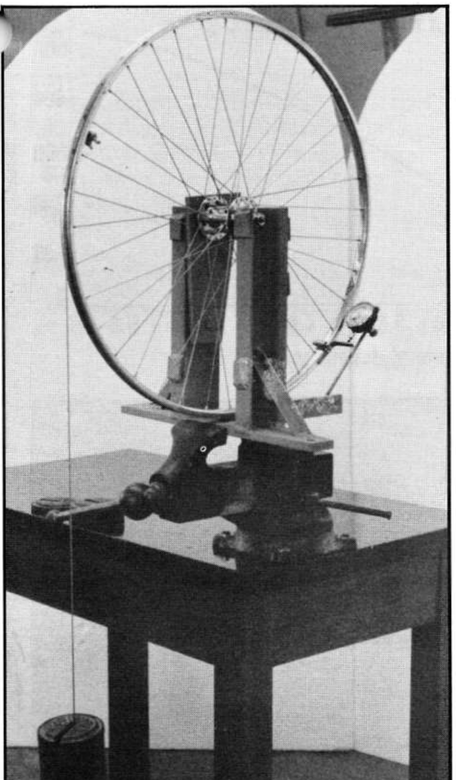


Figure 4: Torsion test apparatus.

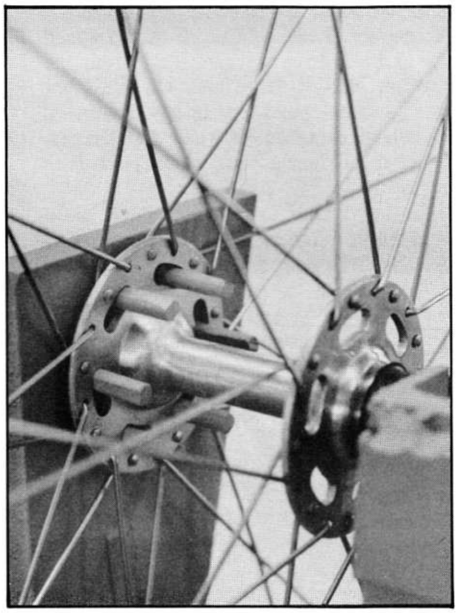


Figure 5: Hub anchorage for the torsion stiffness tests.

Measurements on the radial and one-cross wheels were taken with a dial indicator which could be read accurately to 1/1000 inch. For the two-, three-, and four-cross tests an indicator accurate to 1/10,000 inch was used.

A plot of individual data points from the torque test is shown in Figure 7. The points lie along nearly straight lines, as expected. However, on the last three to five data points (with the greatest applied loads) creep was observed, and thus the accuracy quoted above was not achieved. Nevertheless the overall error was estimated to be less than 1% for all spoke patterns.

For each spoking pattern, the slope of its line on the torque-vs.-rotation graph (Figure 7) is its torsional stiffness. To obtain the best estimate of this slope, we applied linear regression analysis to the data points, excluding the first five and last five points to eliminate errors that could result from these potentially non-linear regions. The results are listed below. Note that the tangentially-spoked wheel (4-cross) was about 23 times stiffer torsionally than the radially-spoked wheel. Also, note a significant progression of increased torsional stiffness with increased spoke crosses.

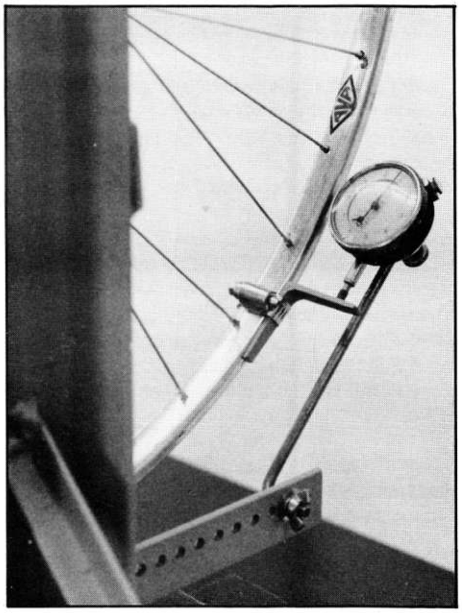


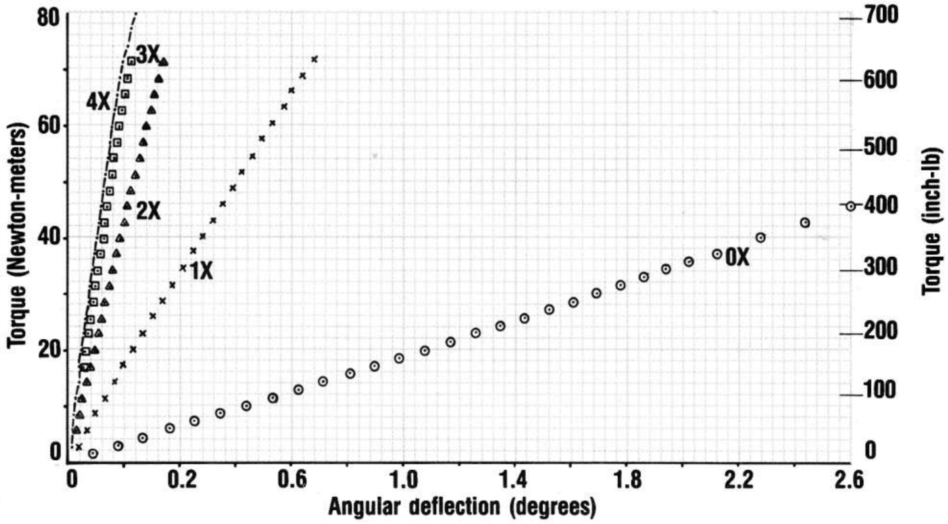
Figure 6: Measurement of rim rotation in torsion tests.

Measured Torque Stiffness[1]

Spoking	(N-m/deg)	(inch-lb/deg)
0X (radial)	16.14	143
1X	81.12	718
2X	210.0	1859
3X	319.1	2823
4X	373.3	3304

[1]*Conversion factor for torque stiffness: (Newton-m/deg) × 8.8508 = (inch-lb/deg)*

Figure 7: Torque vs. Angular Deflection - Experimental Data



Lateral Stiffness

Lateral stiffness is defined as the sideways force required to produce unit displacement of the rim with respect to the hub. Lateral deflection can account for some energy losses during hard sprinting or hill climbing, when the bike is being forced from side to side.

Lateral stiffness is affected by the distance between hub flanges, spoke tension, the number of spokes, and spoke gauge. The distance from the flange to the centerline of the rim is the most influential parameter because it determines the lateral component of spoke forces in the wheel. For this reason, excessively dished wheels (6- and 7-speed rear wheels) have little strength from the sprocket side of the wheel. On the other hand, a three-speed bicycle has the rim of the wheel centered between the flanges, therefore producing equal stiffness on both sides.

For the lateral tests, the wheels were mounted in a fixture with the hub securely fastened from both sides (Figure 8). Loads were applied by the MTS machine to a four-inch arc on the side of the rim at a point located halfway between the valve stem and the rim seam (Figure 9). The resulting force-deflection curves for this test (Figure 10) are nearly straight lines when lateral force is in the range of 20 to 80 lb. We measured the slope of these lines in this range, and report the results as lateral stiffness:

Measured Lateral Stiffness[2]

Spoking	(N/mm)	lb/inch
0X (radial)	107.8	616
1X	114.8	656
2X	111.0	634
3X	106.2	606
4X	100.9	576

Note that the wheels with shorter spokes are slightly stiffer than wheels with longer spokes, except for the radially spoked wheel. This unexpected lower stiffness for the radially spoked wheel could be due to the fact that the outermost crossing in the 1X to 4X spoke patterns is interlaced. The implication is that interlacing of the spokes allows loads to be distributed more uniformly during severe wheel loading.

Radial Stiffness

Radial stiffness is the force required to displace the rim a given amount radially with respect to the hub. This property determines the amount of road shock that the wheel can absorb and is affected by the number, thickness, and tension of the spokes.

Hub flange diameter also plays a role in

[2] *Conversion for lateral and radial stiffness: (N/mm) × 5.7100 = (lb/inch)*

Figure 8: Lateral stiffness test fixture (disassembled).

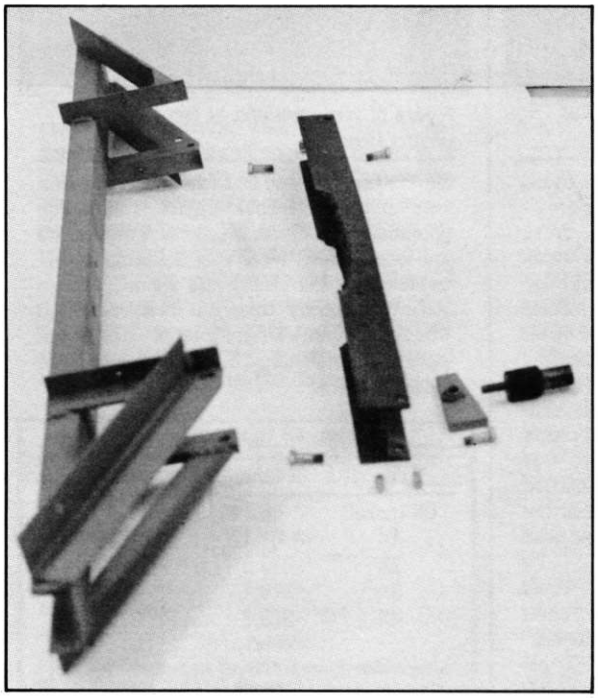


Figure 10: Lateral Force vs. Deflection - Comparison of all spoke patterns

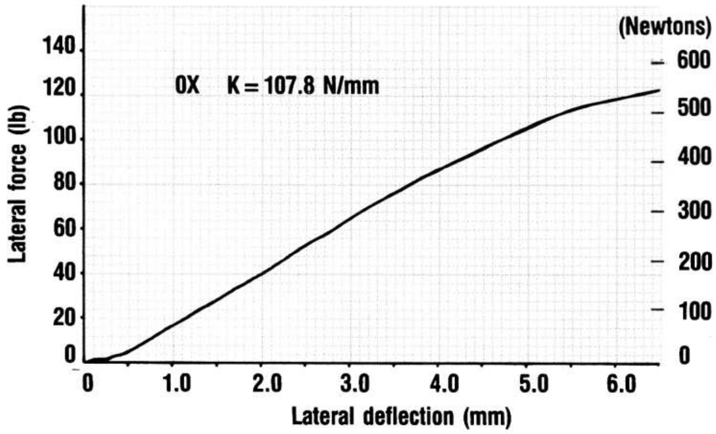
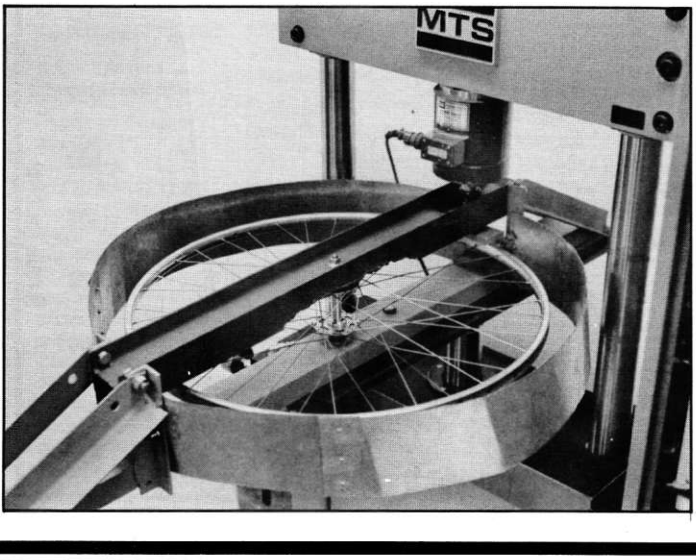
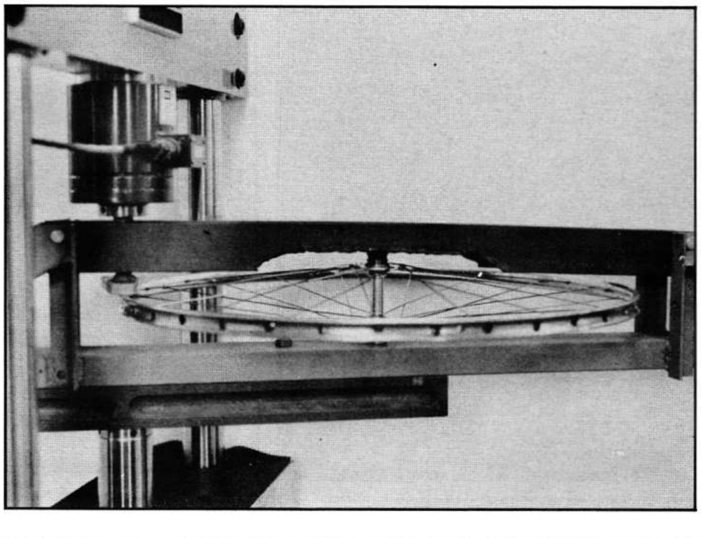


Figure 9: Lateral stiffness test fixture in MTS machine.





wheel stiffness. European road racers usually employ small flange hubs for a smoother ride on their traditionally rougher road surfaces. For the repeated cornering and acceleration requirements of criterium and track racing, large flange hubs provide greater lateral and torsional stiffness.

For the tests, the wheels were mounted in the vertical test stand (Figure 11). A radial force was applied to the rim through a fixture which distributed the load over a four-inch arc (Figure 12), since this approximates the length of tire in contact with the road surface. For all wheels tested, the load was applied halfway between the valve stem hole and the rim seam. The speed of the MTS machine was set to produce a displacement of 1.4 millimeters/minute. This slow loading rate eliminated errors that dynamic loading conditions could create. The force-deflection curves are plotted in Figure 13, and the slopes from the linear range (100 to 200 lb) are as follows:

Measured Radial Stiffness[2]

Spoking	(N/mm)	lb/inch
0X (radial)	2444	13,960
1X	2075	11,850
2X	2475	14,130
3X	2286	13,050
4X	2096	11,970

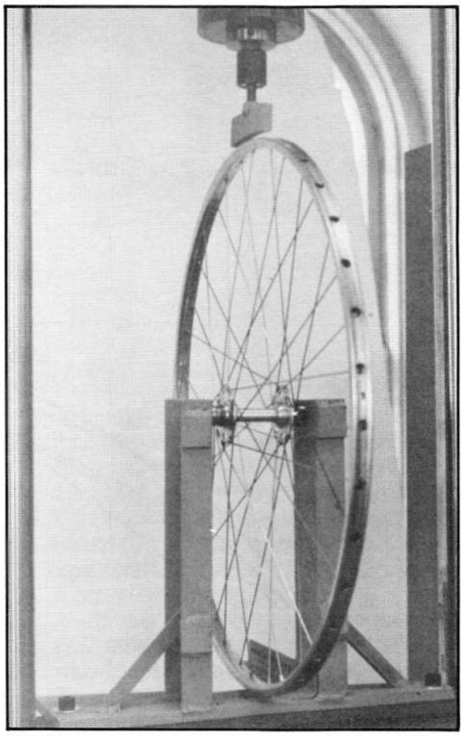


Figure 11: Radial stiffness test apparatus.

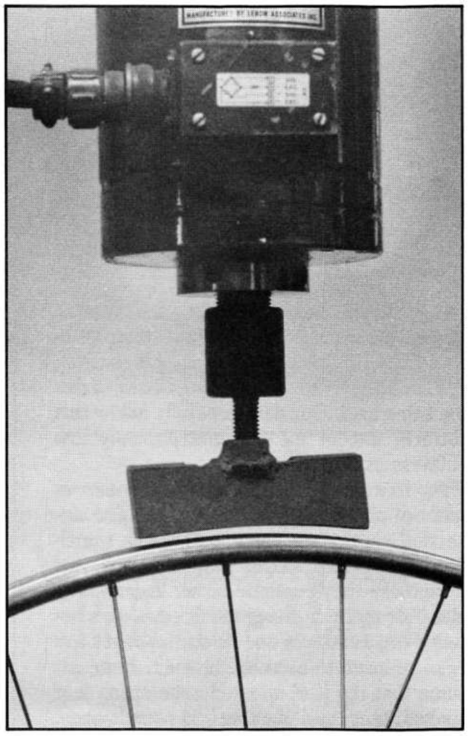
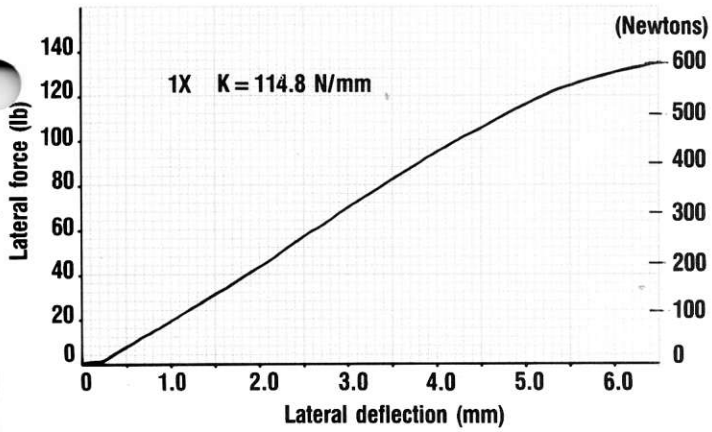
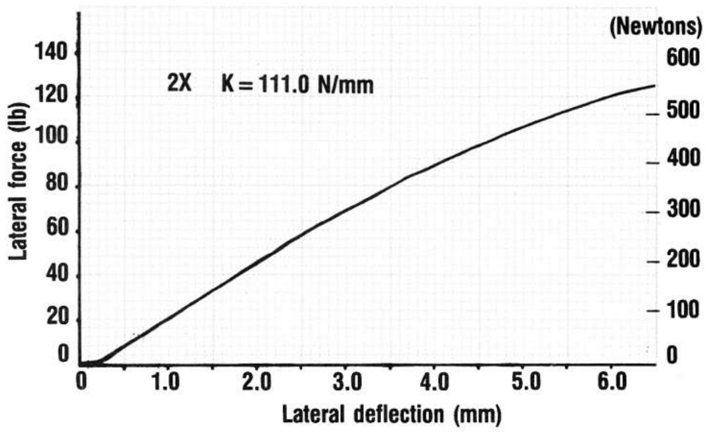
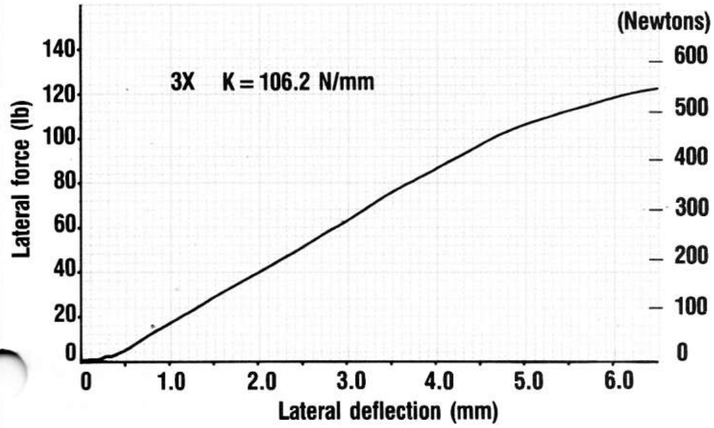
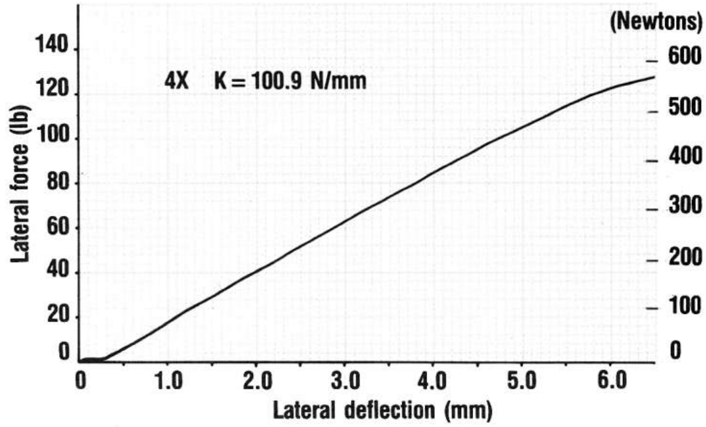


Figure 12: Detail of radial loading applied to wheel.









One would expect wheels with shorter spokes (fewer crosses) to be stiffer than wheels with longer spokes, since long spokes stretch more than short ones under the same load. Our data generally follow this pattern, except for the unexplainably low stiffness of the 1X wheel.

An interesting phenomenon happens at loads of about 400 to 500 pounds. The deflection curves are linear below this transition region; their slopes then decrease, and from there they continue rather linearly. It's plausible that this change in slope occurs because the tension in one or more spokes has been reduced to zero by the load. If we assume that the load-affected zone spans four spokes, tensioned to about 136 pounds each, and ignore any stiffness contributed by the rim, the wheel could theoretically support a 544-pound load with no spoke going slack. This load is fairly close to the 400-500 lb. transition zone seen in Figure 13.

Even more interesting is the stiffness curve at extremely high loads (Figure 14, for a 3X wheel). At about 800 lb., a second transition zone is seen, and the stiffness decreases even further. At this point, the stiffness is only about one-twelfth of its value under light loads. It's possible that this is due to further unloading of the downward spokes with major redistribution of forces through deformation of the rim. In any case, the implication is that the wheel's response to high shock loads (vibration and road impact) is not a simple function of its resistance to relatively small (and static) gravity loads. Further testing to verify these ideas would be valuable.

We draw the following overall conclusions from these tests:

- —The property most affected by spoking pattern of a wheel is torsional stiffness. In comparison to the 4X wheel (which is stiffest, as expected), the 3X wheel is about 85 percent as stiff, while the radial wheel is only 4 percent as stiff.
- —Lateral stiffness is only slightly affected by spoking pattern, with the shorter-spoked (fewer cross) designs being up to 15 percent stiffer than longer-spoked ones.
- —Radial stiffness shows a similar pattern as lateral stiffness, but our data are somewhat inconclusive due to a low stiffness measurement on the 1X wheel.
- —Radial stiffness decreases greatly at high loads. Thus, the wheel's response to road impact may be hard to predict from the steady-state properties tested here.

Figure 13: Radial Force vs. Deflection - Comparison of all spoke patterns

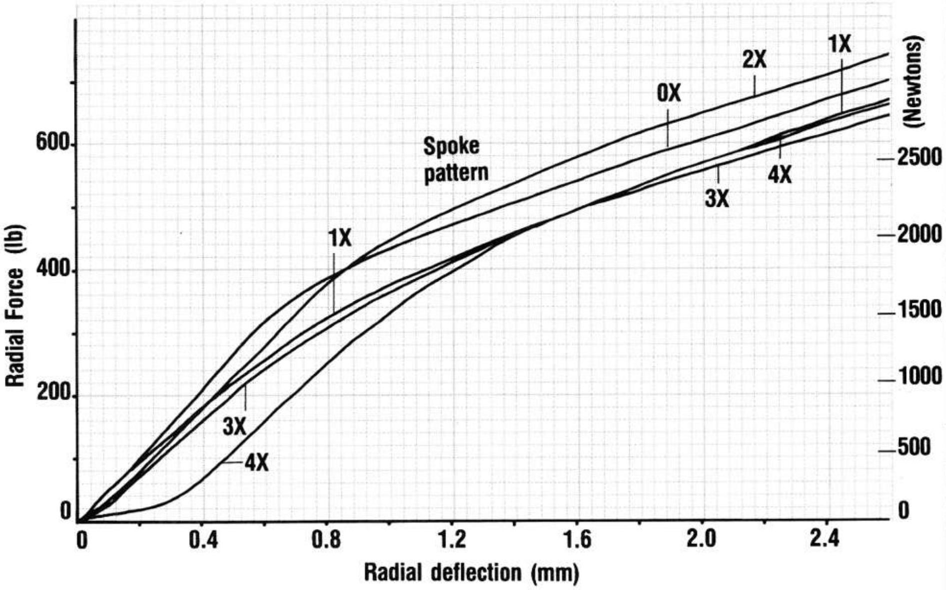
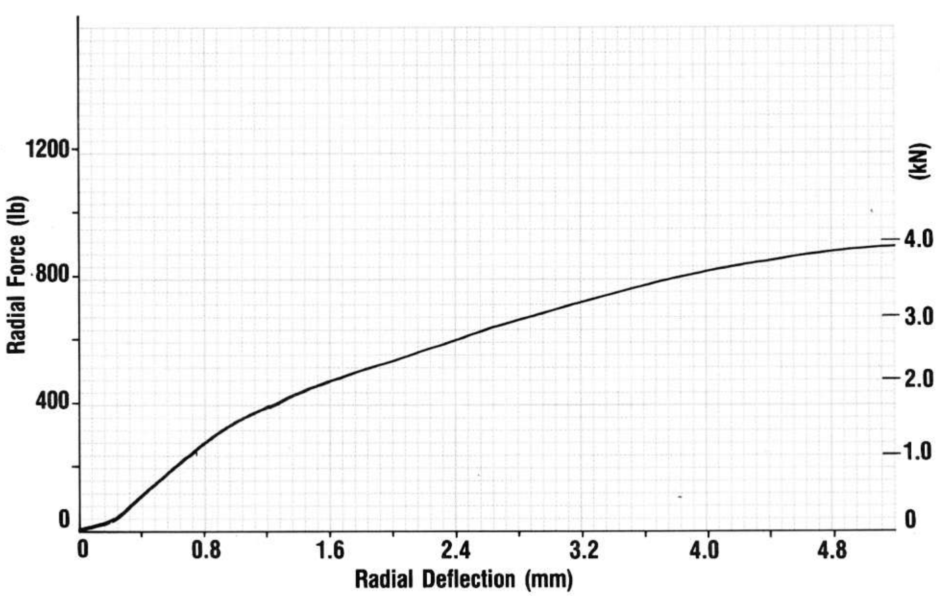


Figure 14: Radial Force vs. Deflection at High Forces - Three-cross spoke pattern



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—Archibald Sharp, *Bicycles and Tricycles*, (Cambridge, MA: MIT Press, 1977), pp. 337-346, 351.

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—Dept. of Engineering Science & Mechanics and Engineering Research Institute for use of laboratory facilities, test equipment and support for one of the authors (A.A.).

COMPARISON OF WHEEL STIFFNESS DATA		SPOKE PATTERN					
	INVESTIGATOR	0-cross	1-cross	2-cross	3-cross	4-cross	SOURCES OF DATA
Torsional Stiffness (inch-lb/degree)	Price & Akers (tests)	143	718	1859	2823	3304	article above
	Brandt (calculations)	—	—	—	2118	—	Bicycle Wheel, p. 136
	Brandt (finite elements)	—	—	—	2094	—	Bicycle Wheel, Fig. 70, p. 144
	Goldberg (calculations)	0	1088	3594	5923	6856	Spoking Word, Table K-2, p. K-19
Lateral Stiffness (lb/inch)	Price & Akers (tests)	616	656	634	606	576	article above
	Brandt (tests)	—	—	—	200	—	Bicycle Wheel, Fig. 17, p. 42
	Goldberg (calculations)	1430	1401	1325	1222	1115	Spoking Word, Table K-3, p. K-23
Radial Stiffness (lb/inch)	Price & Akers (tests)	13,960	11,850	14,130	13,050	11,970	article above
	Brandt (finite elements)	—	—	—	18,281	—	Bicycle Wheel, Fig. 67, p. 141
	Goldberg (tests)	14,500	—	—	—	—	Spoking Word, Table G-2, p. G-15

Wheel Stiffness: Theory Meets Experiment

The test results of Price and Akers, from the article above, can now be compared against the calculations set forth by Jobst Brandt (in *The Bicycle Wheel*, 1983 revised edition) and by Leonard Goldberg (in *The Spoking Word*). The agreement, as you'll see below, is notably poor in some areas, and good in others. The bottom line of this comparison is that we now know the limitations of these two useful books a little better.

The table in this sidebar lists data on the three varieties of wheel stiffness (torsional, lateral, and radial) from the three studies mentioned above (Price/Akers, Brandt, and Goldberg), for five spoking patterns (0-cross through 4-cross). From Brandt's and Goldberg's books, I extracted whatever numerical data even remotely pertains to wheel stiffness. Their data is sparse, as seen by gaps in the table. Brandt does not actually give numerical values for stiffness, but instead quotes forces and deflections, which were ratioed and converted into customary English units to obtain the stiffness numbers in the table. Some data from both Brandt and Goldberg are from actual tests they performed; these are identified in the table.

Wheels used in the three studies are of *similar*, but *not identical*, construction. They all have 36 straight-gauge spokes; Price's and Goldberg's were 1.8 mm in diameter (15 gauge), while Brandt's were 1.6 mm. Spoke tensions were: 136 lb. for Price, 100 lb. for Goldberg, and unstated for Brandt. Price and Brandt used 700C tubular rims, while Goldberg's rim was a 27-inch clincher. Hubs were: high-flange (63 mm effective diameter) for Price and Goldberg, and small-flange (39 mm) for Brandt.

With all these differences in construction, what can we possibly learn from this comparison? Three patterns emerge after some reflection:

—Brandt's *calculated* stiffness values agree well with the Price/Akers test results. Example: for the 3-cross design, Brandt calculates torsional stiffness in the 2100 in-lb/deg range, versus Price's measured 2823. This is good agreement considering the differences in wheel construction, and encourages confidence in the finite-elements computer model used by Brandt.

—Brandt's only *measured* data point (lateral stiffness of 200 lb/inch for a 3-cross wheel) seems curiously *low* compared to Price/Akers' results in the 600 lb/inch range. This 3-to-1 discrepancy seems like more than can be accounted for by the small-flange/large-flange difference in the wheel constructions tested. Maybe lateral stiffness calls for more care in measurement than either experimenter provided.

—All of Goldberg's calculated values seem *too high*, roughly by a factor of two compared to the Price/Akers tests. Still, Goldberg's calculations show the "correct" patterns (ie, more crosses mean greater torsional stiffness, and less lateral stiffness). The only significant difference between Goldberg's and Price's wheels was the type of rim (clincher vs. tubular), and it is hard to believe that this could explain the consistent 2-to-1 discrepancy in results. Moreover, Goldberg's one *measured* data point (radial stiffness on a 0-cross wheel) agrees very well with the Price/Akers result for this same spoke pattern.

This all suggests that Goldberg's calculations contain a hidden assumption which artificially amplifies his stiffness values by roughly a factor of two. In my opinion, the likely culprit, which incidentally is clearly described in his book, is Goldberg's use of what he calls "effective elasticity of the hub/spoke/rim combination." He uses this one number in an attempt to lump together all of the complex deformations occurring in the wheel (spoke stretch and rim bending being primary). The problem seems to be in the numerical value he assigns to this "constant." For reasons that are unclear, he uses a value that is *twice* as large as what he measured in tests he performed. Had he used his smaller, measured value, all his calculated results would agree much better with the Price/Akers data.

—Bob Flower

SAFETY

Reflectors and Reflective Materials

David Sellers

What factors influence how well bicyclists can be seen by motorists at night? Can reflectors or reflective materials, properly utilized, ensure safety to nighttime riders? In this article we explain how reflectors work, and describe some common reflector products for bicycles. Finally, we take you through the procedure we employed to test 13 different reflective "treatments" under actual nighttime conditions using real people as perceptual observers. While we can't claim to have found the final answer, our results should be of certain interest to anyone who ventures out to share the road with automobiles at night.

Visibility at Night

Recent statistics compiled by the Fatal Accident Reporting System (FARS) of the National Highway Traffic Safety Administration show that, even though the total number of bicyclists killed in traffic accidents has decreased in recent years, the *percentage* of those fatalities occurring at night has *increased*. 1982 statistics present the most dramatic results when nighttime fatalities were 42 percent of the total; and this, despite the fact that only a small part of all cycling is done after dark (four percent by one account).

One thing seems certain—these nighttime cyclists are not being seen by motorists as

Dave Sellers is Project Manager in the Rodale Press Product Testing Department. His report on photometric testing of bicycle headlight systems is scheduled for the next issue of Bike Tech.

well as they could be. Greater voluntary use of reflectors and/or lights to render the cyclist more visible could improve this situation significantly.

Actually, for a bicycle to be *visible* to a nighttime motorist is not enough. That is just the beginning of the perceptual response process that an observer undergoes in more or less automatically in the following summarized steps:

1. **Detection**—This corresponds to the first visual sensation that an object is present in the (driver's) field of view. Perceptual tests that measure pure detection (when an object just becomes visible) are called "threshold detection" tests.

2. **Recognition**—In this phase the observer senses and processes more information about the object. It is identified as an object familiar to the observer and to the context it is detected in (e.g., a bicycle or pedestrian on the side of a roadway). Also important in recognition is information about the *position* and relative *motion* of the object. How far away is it when detected, and is it stationary or moving? Is it coming toward or going away from the observer and how fast?

3. **Decision**—Having detected and recognized an object, an observer (driver) must decide on appropriate action, such as making corrective maneuvers to avoid striking the object as it is overtaken (or perhaps that no changes in heading are required).

4. **Action**—Here the decision is put into effect. The steering wheel is turned, the brakes are applied and the mechanical systems of the car follow through with the avoidance maneuvers.

Naturally, the above sequence of events is carried out in reality by complex visual, mental, and physical (perhaps even emotional and subconscious) responses. They may take place in an instant if an impending collision is observed, or over a longer period of time if detection comfortably precedes the necessity for action. In any case, this sequence of events elicits a new found respect for the concept of *reaction time* learned in high school driver education class.

Conspicuity is the term given to the collection of attributes displayed by an object which render it detectable and recognizable to an observer. Putting a reflector on a bicycle, for example, makes it more conspicuous—increases its conspicuity to a motorist in nighttime riding conditions. The conspicuity of reflectors or lights depends upon a number of factors which can be optimized when considering how they might be designed and deployed:

—*Intensity* (quantity of reflected light)

—*Size* of the reflector surface area

—*Color* of the reflector (red, white, amber, clear)

—*Distribution* of available reflected light across all possible viewing angles

—*Shape cues*—This refers to the intrinsic shape—often the perimeter outline—of an object that renders it partly or wholly recognizable because of learned association among the majority of people. This can be the result of an historical familiarity—as with, say, the placement of headlights on a car, which are positioned to illuminate the road, but also act to inform observers that "a car is coming." Shape cues can also be intentionally designed and deployed such as the universal red triangular reflector that indicates "slow moving vehicle." Presently no active bicycle taillight systems employ distinct shape cues, and in any case, no universal symbolism for "bicycle" exists. This is probably one of the most fruitful areas of development of increased conspicuity.

Perceptual experiments have shown that to attract the attention of a driver who is not anticipating the presence of an object, its brightness must be up to 1000 times greater than that the driver could perceive if his eyes were fully adapted to darkness on a dark night in ideal weather. In other words, to be recognized a light must be *much* brighter than the surroundings. Additionally, there are many things that compete with a conspicuous object for a motorist's attention: bad weather, dirty windshields, glare and reflections from lights from signs, other cars, streetlights, poor vision, fatigue, drug and alcohol effects, and other factors that detract from driver alertness.

Even the best reflectors render a bicyclist at the mercy of the motorist. If a car's headlights are misaimed, covered with dirt, or worse, if the left headlight is burned out, the reflectors may not return enough light to the driver to assure adequate conspicuity.

Furthermore, nighttime eye fixation studies[1] have shown that the initial detection of a bicyclist on the road ahead will most likely occur with the driver's *peripheral* vision rather than his *foveal* (straight ahead) vision. Experiments indicate that the amount of light necessary for threshold detection increases with increasing distance away from foveal vision (both left and right and up and down). At just 10 degrees left in the periphery, for example, the illumination detection thresholds (in a particular experiment) were 2 to 40 times higher than in the fovea, for highly alerted drivers with no other distracting tasks. A high visual workload and information processing level, as for example driving on a busy street in bad weather, further detracts from peripheral detection capabilities. This information strongly supports the necessity for adequate bicycle reflector equipment.

A reflector is a *passive* lighting device—reflecting the light from an overtaking car's headlights back in the direction of the driver, brightly signaling the presence of a bicycle ahead sooner than would be the case without a special reflective treatment.*

While all materials, surfaces and colors reflect light to varying degrees, a *reflective* treatment or device optimizes the reflective effect. Ordinary dark surfaces may reflect as little as 10 percent of incident light. Reflex reflectors may provide up to 1,000 times more light in the direction of the source than diffusing surfaces. A reflector "borrows" incident light so that, in effect, it becomes a (secondary) light source.

The kinds of reflectors we're interested in are called *reflex reflectors* or sometimes *retro reflectors*. Those terms refer to a class of reflectors that always reflects an incident light beam back along the angular direction it came from, into the area very near the original light source (Figure 1). In contrast, most ordinary surfaces display either specular reflection (a "shiny" appearance), or diffuse reflection (a "dull" appearance), depending on the shape, smoothness and other specific material properties.

Reflex reflections can be constructed in basically two different ways:

• **Cube corner reflectors** operate by reflecting light off three mutually perpendicular plane surfaces, which has the effect of returning the incident light toward its source. The three plane surfaces seem to form one corner of a cube (Fig.1A). In practice, many small three-dimensional "cube corners" are nested together, side-by-side in a continuous

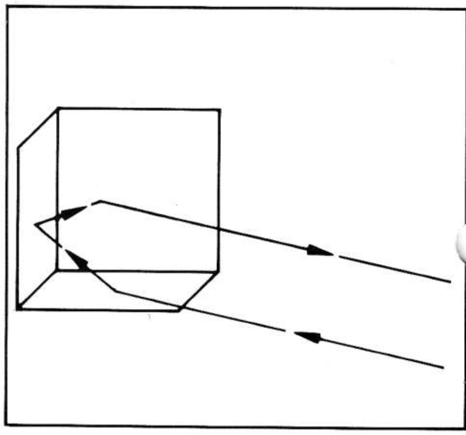


Figure 1a: Light path in a single element of a cube-corner reflector.

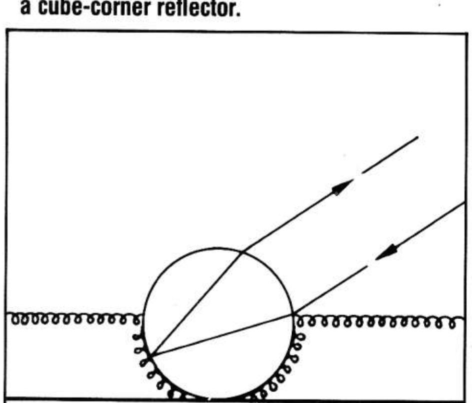


Figure 1b: Single element of half-silvered glass bead reflector.

**The federally-funded NHTSA report on reflectors (see February 1985* Bike Tech*) recommended that cyclists always use an "active source" of light in addition to reflectors.*

pattern over the surface of a reflector—each tiny "cube corner" reflecting light back along the incident path. Cube corners are used in the familiar rigid molded plastic reflectors in red, amber or clear colors, that are used on motor vehicles, bicycles, road signs, et cetera. They can be made cheaply in practically any shape and size.

• **Image-forming reflex reflectors** generally consist of a lens and a reflecting surface working together. Light from a distant source is refracted by the lens, which forms an image at the focal surface. This light is reflected back through the lens along a path approximately parallel to the entrance path and therefore returns light to the source (Fig. 1B). In practice, these lens elements are usually made of tiny half-silvered glass beads. The beads are embedded in a binder or base material and often covered with a transparent film to exclude dirt and moisture. This technology lends itself to producing sheets of material which display the retroreflective property over their entire surface. These "extended surface" sheets can be made very flexible or rigid, depending on the intended end use.

The performance of reflectors depends on obvious factors such as their size, shape, color, whether they are wet or dry, and whether they are clean or encrusted with dirt. In addition, the following two purely optical factors affect the reflector's performance:

—spatial distribution of reflected light: some reflectors concentrate most of the returned light within a small angular distribution (a flat mirror is the most extreme example of this), while other types of reflectors spread the returned light over a wide angular distribution.

—dependence on entrance angle: "Entrance angle" is defined as the angle between an entering light ray and a line that is perpendicular to the reflector's surface (thus an entrance angle of 0 degrees means that the reflector is "lined up" with the incoming light beam). At large entrance angles, beaded reflectors generally perform well, while cube corner reflectors perform poorly. But at small entrance angles (i.e., with a nearly "head on" light beam) cube corner reflectors are superior to the beaded type.

Standardized tests for reflectors used on automobiles, trucks, and traffic signals were developed long ago, and were refined over the years. Recently, some of these standards were expanded to cover bicycle reflectors. The most important of them are:

—Society of Automotive Engineers (SAE) Standard J594-f

—International Standards Organization (ISO) Standard DIS 6742/II

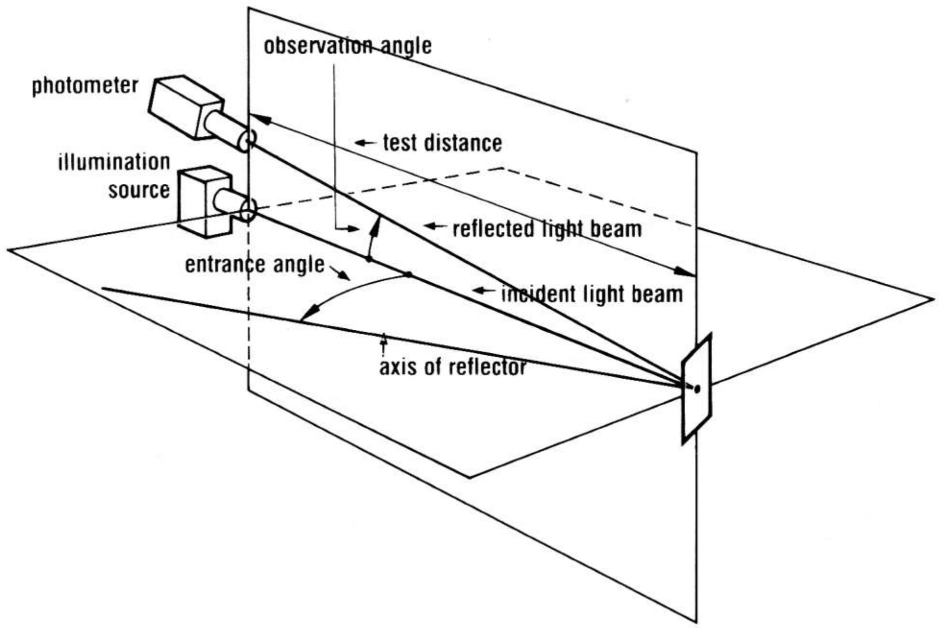
—Consumer Products Safety Commission (CPSC) Standard 1512.16

—US Federal Standard LS-300-C

—Federal Test Method #370

—Federal Motor Vehicle Standard #108

—California Highway Patrol Title 13, Article 14.



Illustrations by George Retseck

Figure 2: Photometric test geometry.

All of these are *photometric* tests; that is, they measure only the intensity of the reflected light under specified conditions of illumination and spatial positioning. The tests proceed somewhat as follows:

The reflector or reflective material is positioned carefully in a test fixture while incident light of known intensity and quality is shined on the reflector. The resulting intensity of reflected light is then accurately measured at several specific angular positions relative to the plane of the reflector.

The two most important terms defining the geometry at the test layout (Fig. 2) are: the *entrance angle*, which was defined above as the angle formed between the incident light beam and a line normal (perpendicular) to the plane of the reflector; and the *observation angle* which is the angle between the incident light beam and the reflected light beam at the observation position (whether the observer is a human eye or a photometric sensor).

The various standards commonly call for photometric readings at observation angles of 0.2 degrees and 1.5 degrees, and entrance angles of 0 degrees, plus and minus 10 degrees vertical and horizontal, plus and minus 20 degrees vertical and horizontal, and other larger angles.

Because the *reflected* light intensity is a function of *incident* light intensity (remember, reflectors don't generate any light of their own), both these quantities must be specified in reflectivity performance ratings. The common unit of reflectivity is candela per incident footcandle (c/f). If the reflector is an extended surface sheet, or if reflectors of significantly different sizes are being evaluated, the *area* of the reflector must be included in the performance rating. Here the common unit is candela per incident footcandle per square foot of reflector area ($c/f/ft^2$). The issue of photometric units and the difference between point light sources and area light sources (as a function of observation distance, reflector size and background illumination) is rather complex. *Reflector Analysis* (Ref. 7) includes a good discussion of this subject.

Some typical reflectivity values for common reflective materials are:

Highway Sign	**70 $c/f/ft^2$**
3M ScotchLite[1] #8910	**450 $c/f/ft^2$**
Reflexite	**250 $c/f/ft^2$**
White Sheet	**0.3 $c/f/ft^2$**

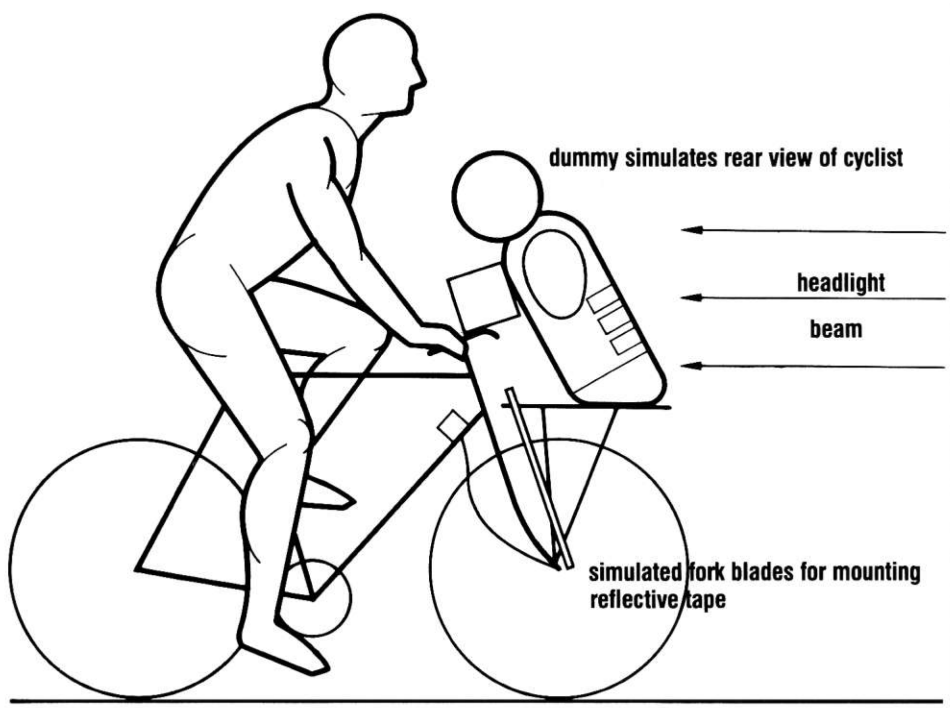


Figure 3: Test bicycle, rider, and dummy used for mounting reflector treatments. Dummy wears: fanny bumper, safety vest, Scotchlite film on black shirt, Scotchlite film on Gore-Tex jacket. Rider wears: reflective tape on helmet, reflective leg bands. Mounted on bike: cube-corner reflectors, pedal reflectors, spoke reflectors, Scotchlite sheet on simulated fork blades.

The reflector standards listed above were established primarily to assist various government agencies in regulating the type of reflectors that could legally be used on the highway. Photometric tests alone, however, do not directly address the question of how a reflector actually performs in the real world, where conditions cannot be so strictly controlled as in a laboratory, and where the subjective factors of human perception and judgment come into play.

Furthermore, the minimum photometric values called out in the standards, as well as the minimal number of angular positions for the measurements, have been criticized as inadequate for safety. In fact, these standards may represent a performance level far below the state of the art capability of reflex reflector manufacturing technology. For all the above reasons, we decided to forego photometric tests of reflectors, and concentrate instead on perceptual field tests. But first a description of reflective products made specifically for bicyclists is in order (See also Figure 5).

Rigid Plastic Reflectors

The *clear and red corner cube reflectors* we tested are typical of those installed on new bicycles by the manufacturer. A clear reflector is usually installed on the front of the bike (facing forward), and the red is installed on the rear (facing backward). These reflectors are available in a variety of shapes: circular, rectangle, shield shape, etc. The net reflective area is generally about 4 to 6 in.^{2} The actual reflective surface of these reflectors is divided into three adjacent planes—the center plane (the largest surface which faces directly backward or forward, depending on where it is mounted), and the two side planes (surfaces that are canted at a shallow angle, one left and one right, from the center plane). This configuration was devised to enable these corner cube reflectors to meet the requirements of the CPSC reflector standard, which specifies performance values at relatively wide entrance angles.

The small *amber corner cube pedal reflectors* are of conventional size (2½ × 5.8″) and are designed to be mounted on the leading edge and trailing edge of each pedal. Their performance is enhanced by their oscillating motion during pedaling.

The eight additional products we tested are all based on the following two reflective sheet materials.

ScotchLite[1]

Scotchlite, by the 3M Company, is a glass bead, exposed lens, wide angle retroreflective sheet. It is made in several forms and colors. In *reflective fabrics,* the tiny glass spheres are bonded to the surface of a durable cloth backing, and the fabric is usually sewn onto garments as trim strips. In *reflective transfer films,* the glass beads are bonded to a film coated with heat-activated adhesive. The reflective surface is protected by a paper or plastic carrier which facilitates handling and application. A removable plastic liner protects the adhesive on the back of the film. The films can be applied using a conventional hand iron.

The fabric and transfer films appear brilliant silver-white when viewed by reflected light at night, and remain highly reflective when viewed at wide entrance angles (though reflectivity necessarily tapers off as entrance angle increases). The fabric and transfer film versions of Scotchlite are distributed by the Safety and Security Systems Division of the 3M Company.

Scotchlite is also available in the form of flexible plastic sheeting with a pressure sensitive adhesive backing. This material, distributed by the Traffic Control Division of the 3M Company, is designed for vehicle markings, roadsigns, etc. The reflective performance is similar to that of the Scotchlite fabrics and films.

Scotchlite was used in four of the reflective products in our perceptual test:

—Glo-Wheel Spoke Reflectors have a tough, Scotchlite film outer surface.

—3M Scotchlite #8170 Bright Silver Reflective Transfer Film was applied to an ordinary black long-sleeved T-shirt. We purposely designed this to represent a *maximum reasonable treatment,* by applying a strip of one inch wide film in a "X" shaped pattern across the back and a horizontal strip at the shoulders and the waist, plus a strip applied to each arm. #8710 film is "elastomeric"—it stretches with the movements of the wearer.

—A *minimum typical treatment* was represented by a commercially available Gore-Tex jacket which included a wide horizontal strip of 3M Scotchlite #8710 Fabric across the back at about armpit height.

—To test the use of Scotchlite on the bicycle itself, we made a fixture to simulate the two seat stays of a bicycle as seen from the rear (Fig. 3). To each slanted, vertical stay we applied 3M #580-10 white reflective sheeting.

Reflexite[2]

Reflexite is a proprietary material which contains reflective elements of tiny plastic cube corner prisms (like the glass beads, but cubic instead of spherical) which are integrally molded into a UV-stabilized vinyl film backing. This material can be plain or adhesive backed, semi-rigid or flexible, and is available in several standard colors. The product seems strong, durable, and impact resistant. It exhibits a smooth outer surface which (according to the manufacturer) makes it easy to clean, has good weather-

[1]Scotchlite *is a registered trademark of 3M Corporation.*

ability, and helps maintain a high level of reflectivity when wet. Reflexite was used in the following products in our test:

—Reflective Helmet and Bike Tape is a set of adhesive-backed, stick-on shapes pre-cut from a Reflexite flexible sheet. For the perceptual test, these Reflexite "stickers" were applied to a bike helmet worn by the test rider, with the majority of the total reflective area toward the front of the helmet facing the light source.

—Reflective Cyclist's Safety Vest is made of light, flexible nylon mesh trimmed with a substantial amount of Reflexite strips front and rear. The vest is put on over the head (like a poncho) and fastens at the sides with Velcro straps.

—Reflective Leg Bands are felt-lined, flexible straps, covered with Reflexite, to be worn encircling the ankle and fastened by Velcro. The leg bands double as trouser protectors.

—Fanny Bumper is a large triangular piece of perforated nylon fabric with a wide Reflexite strip around its perimeter. The fanny bumper is tied around the cyclist's waist with a cord, and is meant to be positioned low on the rider's back, so it presents a nearly vertical orientation.

We also tested two of the reflective products under modifications. Extra trials of the Scotchlite on black shirt treatment and the Fanny Bumper (Reflexite) treatment were run in which they were sprayed with water before the observation. There was a total, then, of 13 reflective treatments evaluated in the perceptual test.

[2]Reflexite *is a registered trademark of Reflexite Corporation.*

Perceptual Field Testing

Our perceptual field tests were designed to measure actual human responses to different reflectors in a realistic setting.

In an effort to keep the tests on a manageable level, we restricted the number of variables tested to the minimum needed to provide a valid comparison. The independent variable is the presentation of the different reflective treatments mounted on a bicycle or rider in a way that simulates the typical use of the reflectors in real life. The response is the visual threshold detection of the reflectors by observers. This approach enables direct comparison of all types of reflective treatments, automatically encompassing the effects of size, color, mounting position and motion (or lack of motion) on visibility.

We ran the perceptual tests on a 1,500 foot straight and level section of isolated roadway (Fig. 4)[3]. Three observers were seated in a stationary car at one end of the roadway. Reflector treatments were mounted one at a time to a bicycle and/or rider (Fig. 3) which then moved slowly towards the observer car from the opposite end of the roadway. The test technicians communicated by portable walkie-talkie radios. The car's headlights were adjusted to conform to Pennsylvania Motor Vehicle Code standards, and were turned on low beam throughout the test. The three observers, seated in the front seat, were instructed to signal when their visual sighting of the reflector was quite certain. Written instructions were issued to the observers outlining the criteria for visual detection.

[3]*Thanks to Mr. Richard Gibson of the Mack Truck Engineering Development and Test Center, Allentown, PA, for use of the roadway test facility.*

During a test run, the bicycle slowly approached the car until each observer, in turn, signaled their observation by hand signals to a worker in the car's back seat. This worker relayed the signals to another worker outside the car by way of a small light mounted on the top of the car. The light could be blinked on by a hand held switch operated by the first worker inside the car. The second worker relayed this signal, via walkie-talkie to the rider on the bicycle. Each time the rider received a signal, he recorded the distance traveled (for that run) from a small accurate on-board counter driven by wheel revolutions of the bicycle. Calculations were made later to figure the true detection distances.

Two complete trials of each of the thirteen reflector treatments were completed on each of two consecutive nights, utilizing the same three observers for all the trials. Thus, a total of 156 separate observations were recorded.

We took considerable pains to have the reflective treatments presented to the observers as they would appear when the car is overtaking the bike from behind. The corner cube reflectors, the Scotchlite on simulated seat stays, the pedal reflectors, spoke reflectors, helmet tape and leg bands were all mounted in the usual manner, with the actual front viewed observation simulating a rear viewed observation fairly accurately. Other reflective treatments normally worn by the rider were positioned to simulate the particular height and angle of the rider's torso as it appears to an overtaking car from behind. To

Figure 4: Plan view of field test site.

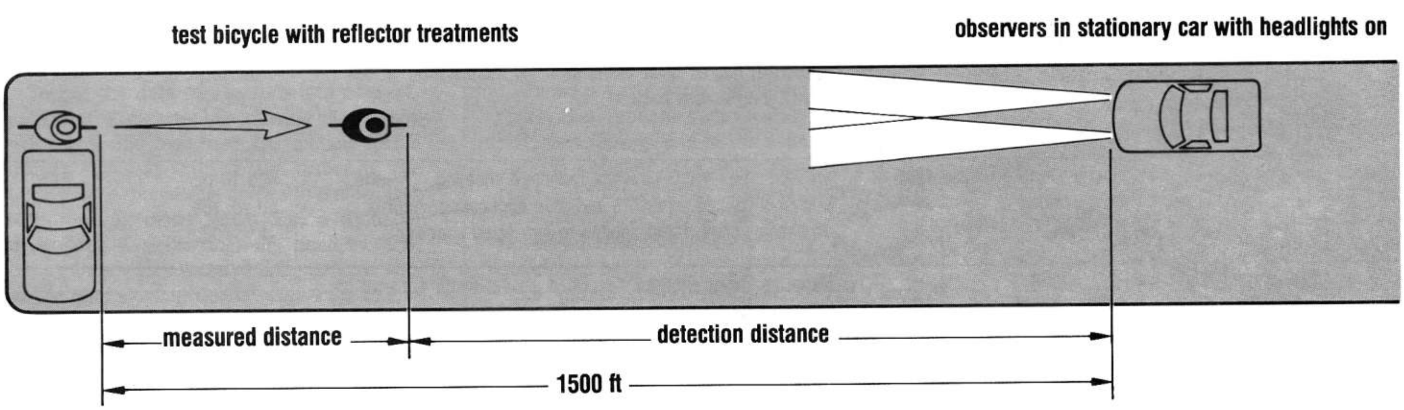


Figure 5: REFLECTORS AND REFLECTIVE MATERIALS

Reflector	Manufacturer or Source	Size and Description	Cost	Detection Distance (ft) in Perceptual Test
1. Clear Corner-Cube Reflector (From Kit #800NS)	Sate Lite Mfg. Co. 6220-30 Gross Pt. Rd. Niles, IL 60648	Conventional 3-sided, front mounted reflector, approx. 4 in^2, nearly rectangular shape.	$.50-1.00 each	1264
2. Red Corner Cube Reflector (From Kit #800NS)	Same as 1.	Conventional 3-sided rear mounted reflector, approx. 4 in^2. Nearly rectangular shape.	$.50-1.00 each	990
3. Two small amber corner cube reflectors, pedal mounted. Model #RR-0217-EZ-1A	Cat Eye brand, made in Japan, distributed by: West Coast Cycle 8631 Hayden Place Culver City, CA 90230	Conventional pedal-mounted, molded plastic reflectors (2 on each pedal) approx. 1.5 in^2. Rectangular shape. Package of four for 2 pedals.	$2.50-4.00 per package	815
4. Glo-Wheel Spoke Reflector (Scotchlite)	Cycle Components P.O. Box 4363 Fullerton, CA 92634	Small, flexible, cylindrical, "Scotchlite" covered reflectors 1/4" diameter × 2 7/8" long. Slit lengthwise to fit over spoke. Pkg. of 8 for two wheels.	$3.98/pkg.	560
5. Reflective Helmet and Bike Tape Item #2007-HBT (Reflexite)	Bike-A-Lite Box 125 Silver Lake, NH 03875	Adhesive backed, stick-on shapes, includes circles, rectangles, and triangles, approx. 36 in^2 total area. Applied to helmet for our test.	$2.49/pkg.	376
6. Reflective Cyclist's Safety Vest Item #2002-CSB (Reflexite)	Same as 5.	Flexible fluorescent orange mesh vest with 2 full length Reflexite strips (1 inch wide) vertically front and rear, and one full width Reflexite strip horizontally rear (1 1/2 inch wide). Approx. 82 linear inches Reflexite.	$9.95	369
7. Reflexite Leg Bands (2) Item #2006-LB1 (Reflexite)	Same as 5.	Flexible band of felt-lined Reflexite, 1 1/2" wide × 14" long. To be strapped around ankle and fastened with Velcro. Sold one per pkg.	$2.29 each	477
8. Fanny Bumper Reflexite	F. B. Action 824 S. Remington Rd. Columbus, OH 43209	Fluorescent orange polyurethane coated, perforated nylon fabric in triangular shape (13 inches/side) with one inch wide Reflexite border. Enamel coated spring steel stiffeners on two sides.	$4.50 each	890
9. Scotchlite Reflective Transfer Film on Black Shirt (Maximum Treatment)	Safety and Security Systems Divisioln/3M 223-3N 3M Center St. Paul, MN 55144	3M Scotchlite #8710 bright silver reflective transfer film 1" wide. Approx. 70 linear inches applied to back of shirt and 18 linear inches applied to each sleeve of shirt.	$27.10 per 50 yd. roll, 1 inch wide.	808
10. Scotchlite Reflective Transfer Film on Gore-Tex Jacket (Minimum Treatment)	Performance Bicycle Shop P. O. Box 2741 Chapel Hill, NC 27514	"Performance" Gore-Tex rain jacket with one horizontal Scotchlite #8910 silver fabric strip; 3/4 inches wide × 19 inches long across back.	$94.95 (Jacket)	661
11. Scotchlite Reflective Sheeting, 1" Wide on Seat Stays	Traffic Control Division/3M 223 3M Center St. Paul, MN 55144	3M Scotchlite #580-10 white reflective sheeting, 1" wide, adhesive backed, pressure sensitive application. Approx. 17 linear inches (total) applied to simulated seat stays.	$23.10 per 50 yd. roll, 1" wide.	617
12. Fanny Bumper (wet) (Reflexite)	Same as 8.	Same as Fanny Bumper described above; water sprayed on reflective surface before test trial.		796
13. Scotchlite Reflective Transfer Film on Black Shirt (wet)	Same as 9.	Same as Scotchlite Transfer Film described above; water sprayed on reflective surface before test trial.		839

accomplish this, we constructed a life-size dummy (head, torso and hips) which we mounted on the front of the bicycle, with its back facing forward (see Figure 3). This allowed the bicycle to be pedaled forward (a real convenience for the human rider) while presenting a realistic simulation of a rider's backside to the observers. The dummy was covered with black cloth and the human rider wore dark clothing. A general effort was made to ensure that no stray light from random reflections or other sources would lead to erroneous observations. The "wet" Reflexite and Scotchlite trials were accomplished by spraying the treatments, as mounted on the dummy, with a water spray bottle immediately before the rider began these test runs. The order of all the treatments was randomized by drawing from a deck of shuffled playing cards.

Test Scoring and Results

Each reflective treatment received a total of twelve responses (four trials for each of three observers). The arithmetic means of the 12 responses represent the detection distance scores (in feet) for the treatments, and are listed in Figure 5.

The best reflective treatment in this test was the clear corner-cube reflector, with a detection distance of 1264 feet, while the least effective treatment was the cyclist's safety vest, with a detection distance of 369 feet.

A statistical analysis of variance was performed to obtain a measure of significance for the differences between treatment means: Any two treatments differing by more than 151 feet are found to be significant at the 90% confidence level. In other words, if two detection distance scores differ by more than 151 feet, we can be 90% certain that the difference is because one reflector really is better than the other one. If the scores differ by less than 151 feet, we cannot tell whether the difference is due to random variations in the test trials or due to true performance differences in the reflectors. Thus, if two reflectors differ in detection score by less than 151 feet, additional test trials would be needed to determine which one is "really better."

To put the data of Figure 5 into context, note that an automobile moving at 55 mph will travel 550 feet from the instant that a danger signal appears to the driver to the point where the vehicle comes to a stop. It is true that, in many situations, a motorist overtaking a cyclist does not need to stop, but can take other evasive actions (swerving to the left, slowing down). Nevertheless, it is not reassuring to find that the detection distances for many of the reflective treatments we tested are less than, or roughly the same as, the motorist's stopping distance from 55 mph.

The test results show that some reflector treatments are clearly more effective than others. The standard clear and red corner cube reflectors were the best performers. The Scotchlite on black shirt (maximum treatment) and Scotchlite on Gore-Tex jacket (minimum treatment) show a difference that is just on the border of the significant difference range—the maximum treatment is probably a better performer but not overwhelmingly so.

The test failed to show a significant difference between dry and wet treatments with the Fanny Bumper (Reflexite) and the Scotchlite on black shirt. We did not monitor the "wetness" of the treatments: it is entirely possible that the water film evaporated to some degree by the time they were detected by the observers. Therefore, the information given by our "wet" trials is inconclusive.

We believe that a generalization can be drawn about performance relative to the location of the reflective treatments on the bicycle. Treatments appear to be less effective if they are mounted higher on the bike or rider, or if they are slanted at an angle from the vertical. The helmet tape, which is located at the highest possible position, performed poorly. The reflective cyclist's safety vest (which we expected to perform well) performed poorly, presumably because it is located fairly high and because it was oriented at a 45 degree angle from the vertical (just as it would be on the back of a rider bent over the handlebars); the larger entrance angle undoubtedly contributed to the poor performance. In contrast, the Fanny Bumper performed well because it was located lower and was oriented in a more nearly vertical position. Moral: mount reflective treatments as low as possible and don't slant them any more than necessary.

It's important to remember that these tests were limited to a straight-on viewing relationship between observer and bicycle. Reflector performance may change significantly when viewed from various sideways angles or other conditions different from our test set up. For example, the amber cube corner pedal reflectors outperformed the reflective leg bands in our tests, but in an observation from the side, the leg bands would probably be superior. The Glo-Wheel spoke reflectors also would perform well from sideways observation.

We did not attempt to evaluate the deterioration that reflectors can suffer from dust, dirt, weathering, and in the case of Scotchlite and Reflexite, the deleterious effects of repeated washing or drycleaning. These factors could be tested in the future using the threshold detection methodology we have developed here.

For purposes of comparison, we ran a few trials (not part of the main statistical data set) with no reflective treatments at all. The average detection distance dropped to an alarmingly low 188 feet—only about half of the detection distance of the worst reflective treatment or about 1/7 of the detection distance of the best reflective treatment.

As a final statement we will report the observations of the rider of the test bicycle (an accomplished cyclist and bike commuter): "As the observed cyclist in the tests I had the rare opportunity to use a variety of reflective products and to know exactly when I was being seen, and when I was not. It's clear to me now that I had been seriously overestimating my own visibility while riding at night. During some of the trials I suspected that the crew members were failing to relay the signals to me indicating that the observers had seen me). The car's headlights seemed intense to me and allowed me to see myself clearly. I thought I was well illuminated, yet I was actually invisible to the observers. In the future, I certainly won't rely on reflective materials alone when I ride at night."

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6. "Developing Lighting and Reflectorization Standards"; Fred Delong and John Allen; Bike Tech, *Volume 2, No. 4, August 1983.*

7. Reflector Analysis (Report No. DOT/HS-800-302); *Kenneth G. Cook (Century Research Corp.); prepared for US Department of Transportation, National Highway Safety Bureau, Washington, DC 20591, August 1969. Available from National Technical Information Service.*

8. Lighting Equipment and Photometric Tests *(SAE Handbook Supplement HS 34); Society of Automotive Engineers, 400 Commonwealth Drive, Warrendale, PA 15096; 1982, pages 23.63 - 23.64.*

9. ISO Standard 6742/2: Cycles - Lighting and Reflective Devices - Photometric and Physical Requirements - Part 2: Reflective Devices; *International Standards Organization, 1984. Copies are available from American National Standards Institute, 1430 Broadway, New York, NY 10018 for $16 plus $3 postage.*

10. Consumer Product Safety Commission Standard 1512.16: Requirements for Reflectors. *Available in* Code of Federal Regulations, *Vol. 16, Chapter II, Part 1512.16 from US Government Printing Office, Superintendent of Documents, Washington, DC 20402.*

More Optimum Cadence

I was pleased to see the discussion between Boysen and Hammaker in the October 1984 *Bike Tech*. Like many other cyclists, I've wanted to see some hard data justifying high cadences. Boysen's simple experiments were particularly appealing, since they offered a way for individual cyclists to determine their own optimums. The article inspired me to buy a Tunturi ergometer and do my own measurements, which I'll report when I've collected enough data to analyze.

Something about Boysen's interpretation of his data bothered me: his PSHR (power-specific heart rate) seems designed for easy fudging. The minimum PSHR depends on the value chosen for resting heart rate. For example, if his minimum heart rate (53) were used instead of 70 beats per minute in the formula, the minimum PSHR occurs around a cadence of 110 instead of 80-90 rpm. Before stating that a cadence is "optimum," we should decide what we want to optimize. I feel that there are two reasonable interpretations of optimum:

1) For a given power demand, what cadence minimizes heart rate?
2) For a given heart rate, what cadence maximizes power output?

Boysen's data could be used to answer both questions, but only if we fit some equation to the data, since he was not able to make experiments with different cadences at the same power output. (The Tunturi ergometer allows me to make such measurements.) Ignoring the complex curves he drew (which were forced to pass through his 70 beat per minute "fudge factor"), and examining only Boysen's data points, it seems that the data could well be described as a series of parallel lines. That is, we can express heart rate fairly accurately with an equation of the form:

$$\text{rate} = a \cdot \text{torque} + b \cdot \text{cadence} + c \qquad [1]$$

for some constants a, b, and c. In fact, the booklet that came with my ergometer describes a fitness test based on the assumption that heart rate is a linear function of torque at constant cadence. This test is supposedly recommended by the World Health Organization, but the booklet gives no references.

Fitting such a model to the data requires a simple two-dimensional least-square fit. Not having a computer program to do this, I used a pocket calculator to do some one-dimensional fits. I found a set of parameters that fit fairly well. Given the paucity of data, finding a better fit would probably not give much better information. For analyzing my own data, I'll set up a computer program to do proper 2-d least squares. I'll also see if other models fit the data better. The 1-d fit I found for Boysen's data is:

$$\text{heart rate (bpm)} = 2.5\ \text{torque (ft-lb)} + 0.5\ \text{cadence (rpm)} + 50. \qquad [2]$$

At 0 torque and 0 cadence, the predicted rate is 50, very close to Boysen's reported minimum rate. This is somewhat surprising, since that information was not included in the curve fitting, only the 17 data points of the table on page 12. The pulse rate I calculated from Equation 2 is quite close to the measured pulse rate, particularly at the higher powers:

cad. (rpm)	torq. (ft-lb)	power (hp)	heart rates meas.	heart rates calc.
42	1.02	.0082	81.5	74
72	1.02	.014	87	89
90	1.02	.017	94	98
108	1.02	.021	104	107
124	1.02	.024	124	115
72	4.25	.058	91	97
90	4.25	.073	98	106
108	4.25	.087	106	115
124	4.25	.100	124	123
42	10.62	.085	100	98
72	10.62	.146	108	113
90	10.62	.182	117	122
108	10.62	.218	127	131
124	10.62	.251	140	139
42	24.50	.196	131	132
72	24.50	.336	150	147
90	24.50	.420	157	156

Assuming Equation 2 gives a reasonable fit to the data, we can now find various kinds of optimums. To find the optimum cadence for a fixed power, use Equation 2 to express heart rate in terms of a power and cadence:

$$\text{rate} = 13131\ \text{power (hp)}/\text{cadence (rpm)} + 0.5\ \text{cadence} + 50. \qquad [3]$$

Next, set the partial derivative of heart rate with respect to cadence equal to zero:

$$0 = \text{d rate} \big/ \text{d cadence}$$

$$= -13131 \cdot \text{power} \cdot \text{cadence}^{-2} + 0.5.$$

$$\text{cadence} = 162 \cdot (\text{power})^{1/2}$$

For example, the figure on page 3 of the October *Bike Tech* suggests that a tourist traveling at 15 mph needs about 0.1 hp, giving an optimimum cadence of around 50 rpm, and a heart rate of 101 bpm. A racer going 24 mph needs 0.4 hp for an optimum cadence of 102 rpm, and a heart rate of 152. Note that since cadence increases as the square root of power, torque must also increase as the square root of power. To get higher power most efficiently, you should increase both the force and the speed of pedaling.

Reversing the problem, what if the rider wants the maximum power output for a given heart rate? Rearranging Equation 3 gives:

$$\text{power (hp)} = (\text{rate} - 50 - 0.5 \cdot \text{cadence}) \cdot \text{cadence}/13131.$$

Setting the partial derivative of power with respect to cadence equal to zero gives:

$$0 = \frac{\text{d power}}{\text{d cadence}}$$

$$= (\text{rate} - 50 - \text{cadence})/13131$$

which yields: cadence = rate - 50. Thus for a heart rate of 150, Boysen should get the maximum power output with a cadence of 100. The model fits well at high pulse and power, but not as well for low pulse and power. For example, at Boysen's claimed "resting" heart rate (70 bpm), the model predicts optimum power output of .0152 hp at 20 rpm (a torque of 4 ft-lbs).

Let's examine the dependence of PSHR on Boysen's "fudge factor," according to this model:

$$\text{PSHR} = (\text{rate} - \text{"fudge"}) / \text{power}$$

$$= \frac{13130}{\text{cadence}} + \frac{2626}{\text{torque}} + \frac{5252\,(50 - \text{fudge})}{\text{torque} \times \text{cadence}}.$$

With this approximation, PSHR is minimized for fixed torque when its partial derivative with respect to cadence is minimized.

$$0 = \text{d PSHR} \big/ \text{d cadence}$$

$$= -\text{cadence}^{-2}[13130 + 5252(50-\text{fudge})/\text{torque}]$$

so either cadence = l, or fudge = 50 + 2.5 (and PSHR is flat).

This model predicts that PSHR will decrease with increasing cadence, as long as the fudge factor is less than 50 + 2.5 torque. For Boysen's fudge factor of 70, PSHR should decrease for torque greater than 8, and increase for torque less than 8. With a fudge factor of 50, PSHR should decrease for any positive torque. With a fudge factor of 115, PSHR should increase for torques up to 26 ft-lb.

The model does not exactly fit the data, particularly at low power levels, so the curves predicted may not exactly match Boysen's Figure 3. Still, the dangers of a measure so susceptible to "fudging" should be clear. Since the optimum cadences can be much more directly computed, I see no reason to use PSHR for finding optimum cadences.

Plan for my experiments: Preliminary measurements indicate that my knees hurt if I pedal with more torque than about 50 Newton-meters (37 ft-lb), and that 80 rpm at 25 Newton-meters (18 ft-lb) for 9 minutes will raise my pulse to 155 (as high as I care to go).

I plan to make measurements of my pulse after 10 minutes of exercise at specific levels of cadence (20, 40, 60, 80, 100, and 120 rpm) and power (50, 100, 150, and 200 watts). My ergometer's maximum torque limit prevents me from going over 100 watts with 20 rpm, particularly since the unit does not have enough inertia at 20 rpm to get the pedals past top dead center. Judging from Boysen's figures, 80 rpm may be optimal for the 200 watt load, and I may not be able to handle that high a load at other cadences. I plan to take one or two measurements per day (one at low power followed by one at high power). It's not clear how much this will affect the readings.

Sincerely, Kevin Karplus Ithaca, NY

Robert Boysen replies:

I am certain that with the ergometer, Mr. Karplus will obtain better data than I could gather with my less sophisticated exercycle. And I agree that constant-power and constant-heart-rate cursors would have made my article more thorough. But I'm sure Mr. Karplus realizes that my data was limited by the equipment I used. The best I could do is draw a "contour map" of PSHR at various cadences using the specific power data I had. The results appeared to confirm my constant-torque experiments.

I probably should have explained in more detail the use of my resting heart rate of 70, rather than my minimum rate of 53. When Mr. Karplus begins his ergometer experiments, he will soon discover several interesting phenomena.

First, obtaining one data point a day with ten minutes of pedaling will not yield intelligible results. For individuals in fairly good aerobic condition, the heart rate does some very strange bouncing around for the first 10 to 15 minutes of exercise. The accompanying graph, showing my heart rate vs. time during a recent 12-minute exercycle ride, at constant cadence and torque setting, illustrates this. I have no explanation for these phenomena. Perhaps an interested physiologist could comment. The phenomenon is not exclusive with me: a friend thought he was having a heart attack the first time he observed it. Heart rate can drop even into the low 70's with 1/4 hp output during this period!

Second, after the initial 10 to 15 minutes of exercise, the heart rate response to increased power output becomes very regular and steady (also shown on the curve). For this reason, all of my data was obtained during one long session, starting after a 15-minute warm-up period. I checked for the effects of fatigue by repeating several of the earliest data points at the end of the session. The effect was negligible.

Third, once in a warmed-up condition, the resting heart rate tends to "plateau" at a rate well above the minimum (pre-exercise) rate. I rested (zero power output) briefly between each data point and observed that my plateau was 66 to 73 beats per minute, with an average of 70. Thus, rather than being a "fudge factor" as Mr. Karplus implies, the 70 HR at zero cadence is my most repeated and therefore most accurate data point. I've also noted that the duration of exercise determines how long the heart rate will plateau above the pre-exercise rate. After a 20 to 30 minute exercycle session, the plateau lasts five to ten minutes. After my last double century, my at-rest HR was still over 70 the following morning.

In light of these facts, as well as his poor fit to my data, Mr. Karplus' simplified model [Eq. 1 above] is probably not valid. Instead, a more appropriate model is given by the equation at the end of my original article. The data I have collected fit this equation quite well, with the following empirical constants:

$$PSHR = [(0.046\,C)^{2.17} + 2.10\,T] / 5252\ CT$$

where:

C = cadence in RPM

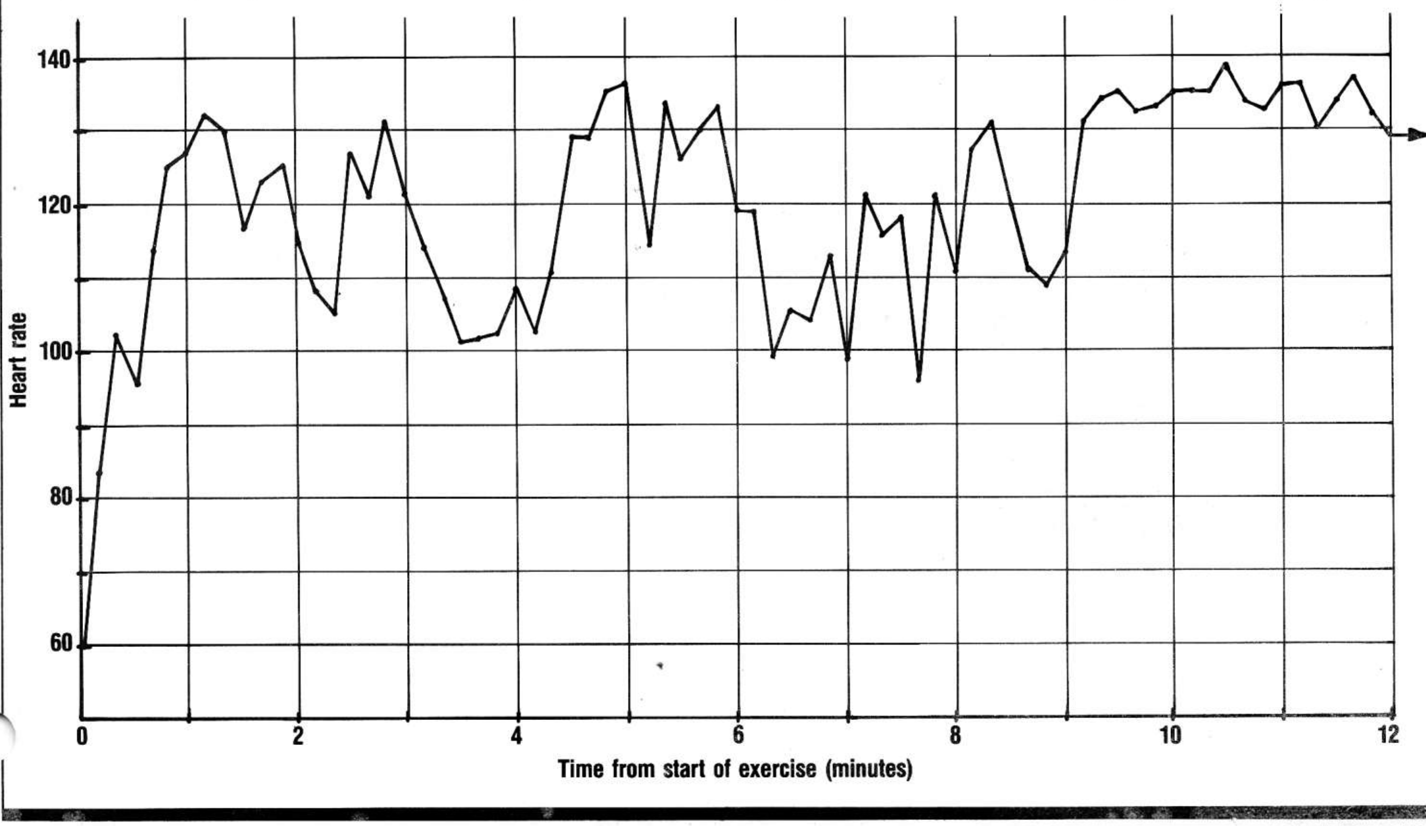
T = torque in ft/lbs

PSHR = beats per minute per unit power output

Setting the total derivative of this equation equal to zero (the condition of minimum PSHR) yields the following expression for optimum cadence: $C^{2.17} = 1431.32\ T$

or, numerically:

Optimum Cadence (RPM)	Torque (ft · lb)
58	5
80	10
115	20



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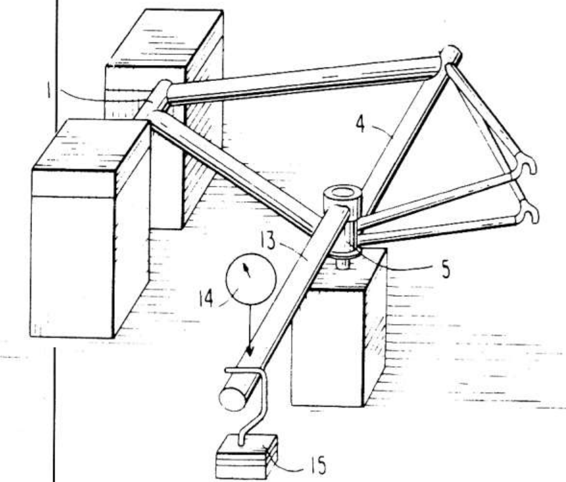
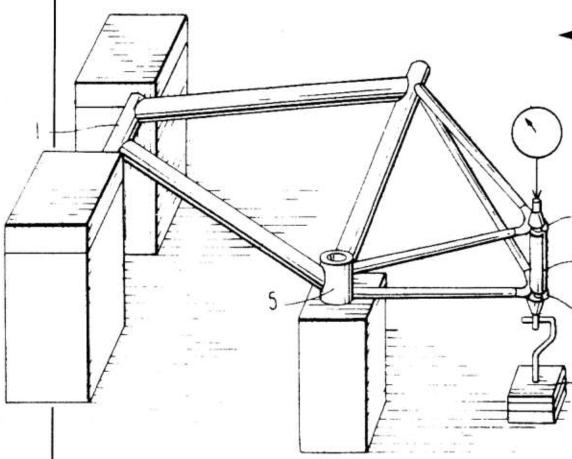




KLEIN FRAMES WIN PATENT: Gary Klein, president of Klein Bicycle Corporation (Chehalis, WA), was granted US Patent No. 4,500,103 on February 19 for his "high efficiency bicycle frame." Klein, one of the first framebuilders in the US to produce aluminum frames commercially, applied for the present patent nine years ago in 1976. The patent covers the weight and rigidity of the overall frame, and the size and rigidity of specific frame tubes. Aluminum alloy 6061-T6 is listed as the preferred material of construction, but other materials (alloys of titanium, magnesium, and beryllium, and high-strength fiber composites) are also listed. The method of joining the tubes is not specifically covered by the patent, but welding followed by heat-treatment to restore strength is mentioned.

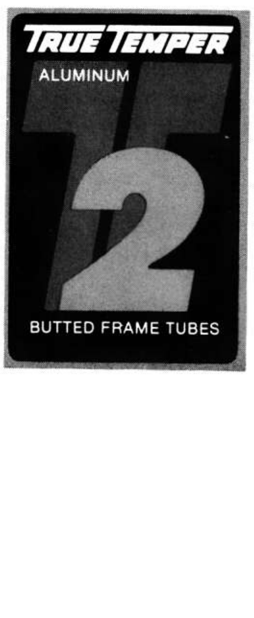
The patent could have a major impact in the marketplace. Aluminum lightweight frames that are relatively rigid are now made by, among others, Cannondale Corp. (Georgetown, CT), Cunningham Applied Technology (Fairfax, CA), and Trek Bicycle Corp. (Waterloo, WI, see item below); Peugeot makes an ultra-light carbon fiber frame (#PY 10 FC). The extent to which any of these frames infringe upon Klein's patent is currently a subject of lively discussion in the industry.

The patent offers an interesting explanation of how a frame can be designed for both high stiffness under pedaling loads (to improve drivetrain efficiency) combined with greater flexibility under "suspension loads" (to avoid a harsh ride).

Klein describes two tests to measure frame rigidity with respect to pedaling loads. First, torsional rigidity at the bottom bracket is determined by clamping the frame to an immovable test bed (see upper illustration at left), and measuring the angular deflection when a weight is applied through a known lever arm distance. The bottom bracket torsional rigidity of conventional steel racing frames is on the order of 42 to 53 foot-pounds per degree, while it is at least 67 foot-pounds per degree for Klein's design, according to the patent.

Second, lateral rigidity at the rear axle is measured by a test in which a weight is hung from the axle and the resulting deflection is measured (see lower illustration at left). Steel racing frames have lateral rigidity on the order of 61 to 75 pounds per inch, while Klein's frames are said to have lateral rigidity of at least 120 pounds per inch.

Klein lists three reasons why his frames provide a "smoother ride" despite their greater stiffness under pedaling loads: First, the frame itself (without front fork or components) weighs less than five pounds, thus reducing the amount of unsprung mass in the bicycle/rider system. Second, the frame geometry has head-tube angles ranging from 73 to 75 degrees and seat-tube angles in the range of 73 to 74 degrees. Third, the seatstays are designed to be relatively flexible; for example, seatstays made of 3/4-inch diameter aluminum alloy tubing of 35 thousandth-inch wall thickness are said to decrease the suspension mode stiffness by "about one-third."



TRUE TEMPER ANNOUNCES FULL LINE OF LIGHTWEIGHT CYCLE TUBING: T-1 chromoly steel tubing and T-2 aluminum tubing is now available from True Temper Cycle Products Division (Memphis, TN), a unit of metallurgical giant Allegheny International Corp. T-1 tubing is produced with yield strengths up to 140,000 psi, the highest rating of any frame tubing today except the upgraded Reynolds 753. T-2 tubing, available in teardrop aerodynamic profile as a special option, was used in building the US "funny bikes" for the 1984 Olympics. Thanks to a solution-heat-treatment and cold-working process developed by True Temper, the yield strength of T-2 is rated 78,000 psi, nearly twice the 40,000 psi rating of conventional 6061-T6 aluminum. The first production bicycle using T-2, the Trek 2000, is assembled using a system of investment-cast internal lugs and adhesive bonding techniques derived from the aerospace program. The bonding process is said to avoid the heat-induced softening that occurs in welded joints.

The **NORTHWEST REGIONAL HUMAN-POWERED VEHICLE RALLY** will be held June 27 - 30 in Seattle, WA, in conjunction with the Seattle-to-Portland Bicycle Classic and the Tour of Puget Sound. In addition to road racing and velodrome-style speed competitions, the HPV Rally will include a public exhibit of the vehicles and two evenings of scientific/technical presentations. For more information, contact Tom McDonald, 110 E. Roanoake Street, Seattle, WA 98102.

ROAD VEHICLE AERODYNAMICS: Second Edition of this 260-page guidebook by A.J. Scibor-Rylski, is now available. The book contains new data on the flows around wheels and wheel cavities, and fascinating photos of the airflow patterns during acceleration and turning maneuvers. Although the book deals with motorized vehicles only, designers of bicycles and HPV's could find this information valuable. ($29.95 from John Wiley, Inc., 605 Third Ave., New York, NY 10157).